PROGRESSION THROUGH CALCULATIONS FOR ADDITION

In Key Stage 1, the principal focus of mathematics teaching is to ensure that **all** children develop confidence and mental fluency with whole numbers, counting and place value. This will involve working with numerals, words and the four operations and will include the use of practical resources. By the end of year 2, children should know number bonds to 20 and use understanding of place value with numbers to at least 100.

In years 3 and 4, teaching will ensure that children become increasingly fluent with whole numbers, the four operations and place value. They will develop efficient written and mental methods and will work on calculations using increasingly large whole numbers.

The main focus for teaching in years 5 and 6 is to ensure that children extend their understanding of the number system and place value using large whole numbers (up to 10 000 000)

By the end of year 6, pupils should be fluent in written methods for all four operations and in working with fractions, decimals and percentages.

Children should not be made to go onto the next stage if:

1) They are not ready.
2) They are not confident.

Children should be encouraged to check their answers after calculation using an appropriate strategy. Children should also be encouraged to consider if a mental calculation would be more accurate and efficient before using written methods.

They are expected to use mathematical vocabulary and to read, spell and pronounce the language correctly.

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies that the children will use throughout their time in school. They are designed to be used alongside written methods.

Mental recall of number bonds

6 + 4 = 10 □ + 3 = 10

25 + 75 = 100 19 + □ = 20

Use doubles

3 + 3 = 6

Use near doubles

6 + 7 = double 6 + 1 = 13

Re-grouping to make 10

6+5 = 11 start with the bigger number and use the smaller number to make 10

6+4 = 10 + 1 = 11

Use partitioning in different ways

47 = 40 + 7 *and* 47 = 20 + 27

Add or subtract 10 from any 2 digit number

46 + 10 = 56 46 - 10 = 36

Addition using partitioning and recombining

34 + 45 = (30 + 40) + (4 + 5) = 79 (start with numbers which do not bridge 10)

Addition using partitioning of second number only

34 + 45 = 34 + 40 + 5 = 74 + 5 = 79

Add 9/11 by adding 10 then adding/subtracting 1

25 + 9 = 34 by 25 + 10 = 35 - 1 = 34

Counting on or back in repeated steps of 1, 10, 100, 1000

86 + 57 = 143 (by counting on in tens and then in ones)

460 - 300 = 160 (by counting back in hundreds)

Add the nearest multiple of 10, 100 and 1000 and adjust

24 + 19 = 24 + 20 – 1 = 43

458 + 71 = 458 + 70 + 1 = 529

Use the relationship between addition and subtraction

Start with low numbers-

3 + 5 = 8 5 + 3 = 8 8 – 5 = 3 8 – 3 = 5

36 + 19 = 55 19 + 36 = 55

55 – 19 = 36 55 – 36 = 19

THE FOLLOWING ARE STRATEGIES THAT WE EXPECT THE MAJORITY OF CHILDREN TO USE AND UNDERSTAND.

INITIALLY:

In play and practical activities children should:

- Begin to make comparisons between quantities, including understanding the meaning of 'more' and 'less',
- Know that a group of objects changes in quantity when something is added or taken away,
- Separate a group of 3 or 4 objects in different ways, beginning to recognize that the total is still the same,
- Find the total number of items in two groups by counting all of them,
- Know that addition is adding two numbers together,
- Relate addition to combining two sets of objects,
- Show an interest in number problems.

THEN:

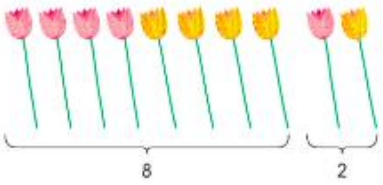
In practical activities (beginning to record when ready) children should:

- Sort/count objects,
- Find the total number of items in two groups by counting all of them,
- Use their own methods to work through a problem,
- Select two groups of objects to make a given total of objects,
- Count repeated groups of the same size,
- Share objects into equal groups and count how many in each group,
- Use a range of vocabulary related to addition, including knowing what 'adding' means,
- Separate a group of objects in different ways knowing that the total is still the same,
- Know and use + sign in practical games and activities,
- Compare two numbers and say which is more or less,
- Compare the amounts in two sets and say which is more or less,
- Say a number lying between two others, or before or after a given one,
- Find one more or less than a number from one to ten,
- Use knowledge of number bonds to do calculations to 20,
- Begin to understand doubling and halving,
- Use knowledge of counting in 2s 5s and 10s,
- Begin to add by counting on from a given number.

Concrete - modelling with real objects

Should we add or subtract to find the total number of flowers?

There are 8 flowers in the vase.
There are 2 flowers in Hannah's hand.
How many flowers are there in total?

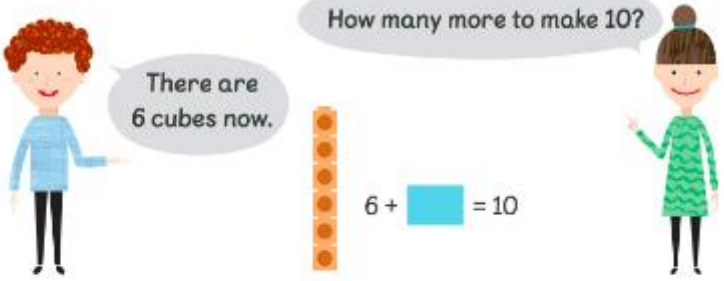


8 + 2 = 10

There are 10 flowers in total.

Concrete - handling real objects

How many more cubes do they need to make a stack of 10 cubes?

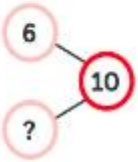


6 + 1 =

6 + 2 =

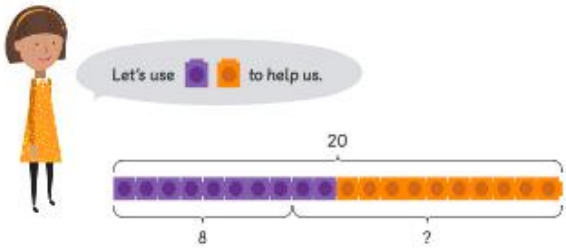
6 + 3 =

6 + = 10



Concrete - modelling with other objects and pictures

Sam bakes 20 cookies.
What if he gives some away?



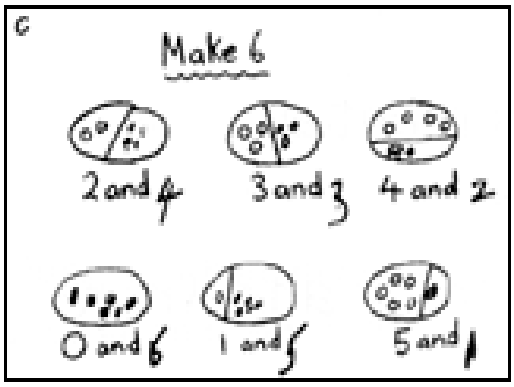
What if Sam gives away 8 cookies?

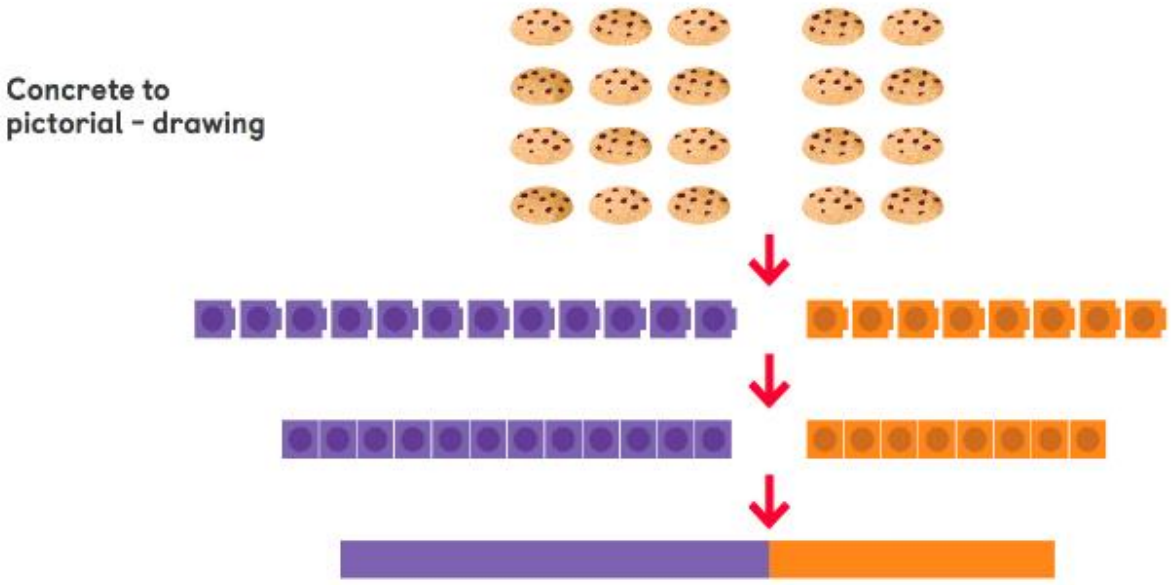
20 − 8 =

Then, Sam would have cookies left.

NEXT:

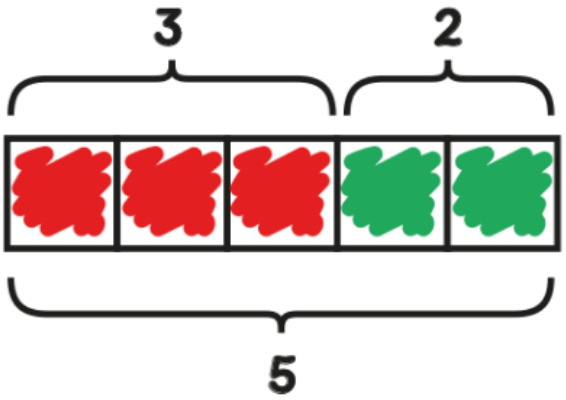
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, e.g.:

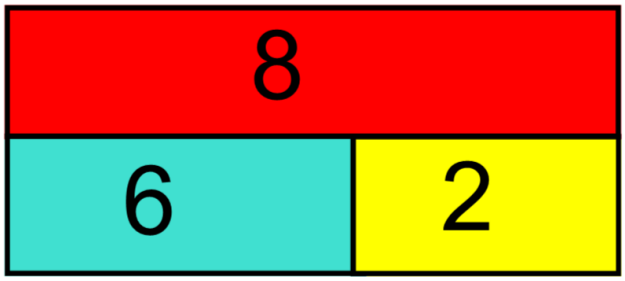




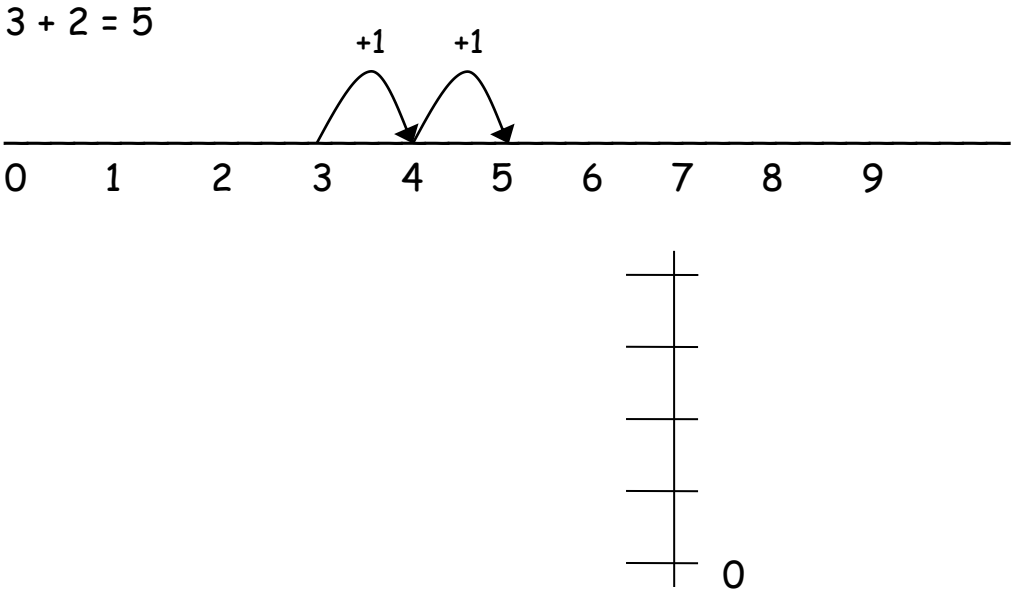
AFTER THAT:

Pupils will naturally develop from handling **concrete** objects, to drawing **pictorial** representations, to creating **abstract** rectangles to illustrate a problem.



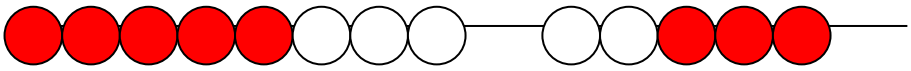


They use number lines (vertical and horizontal), 100 squares and practical resources to support calculation. Teachers must *demonstrate* the use of the numberlines and 100 squares.



Children then begin to use number lines, part/part/whole or bar models to support their own calculations.

For the calculation 8 + 5, bead strings can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3 (e.g. 8 + 2 to make 10, then add 3 more):



Once they are confident in these strategies for adding single digit numbers they will then apply this to adding 2 digit numbers, clearly showing how the tens and units digits change.

Children must understand how to use the knowledge of partitioning to help them when adding:

Start with numbers which do not bridge 10

66 + 22 = ⟶ 60 + 20 = 80

6 + 2 = 8 ⟶ 80 + 8 = 88

Then use numbers which bridge 10

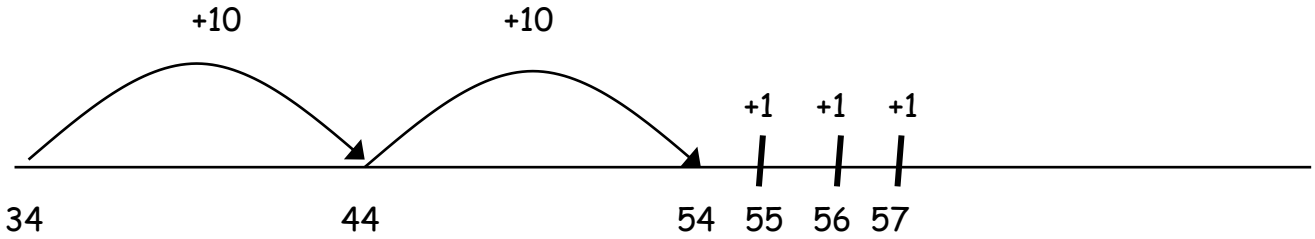
65 + 27 = ⟶ 60 + 20 = 80

5 + 7 = 12 ⟶ 80 + 12 = 92

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on:

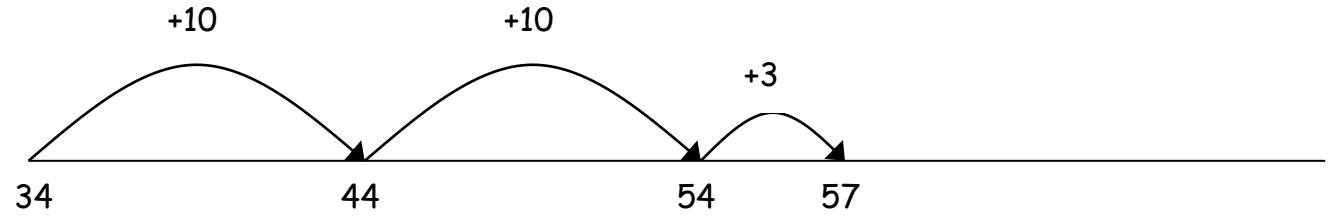
First counting on in tens and ones.

34 + 23 = 57



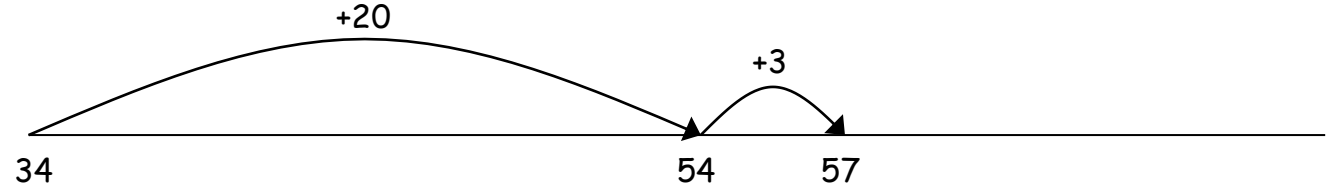
Then becoming more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

34 + 23 = 57

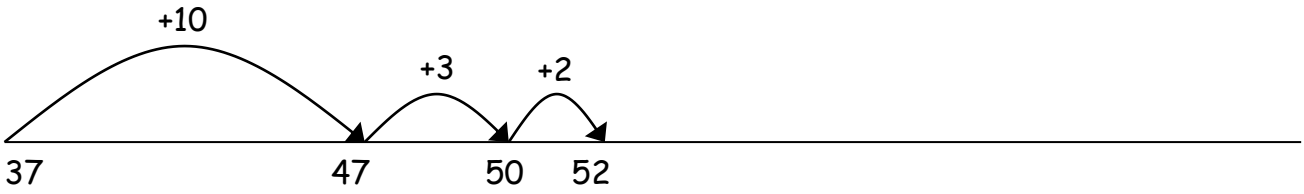


Followed by adding the tens in one jump and the units in one jump.

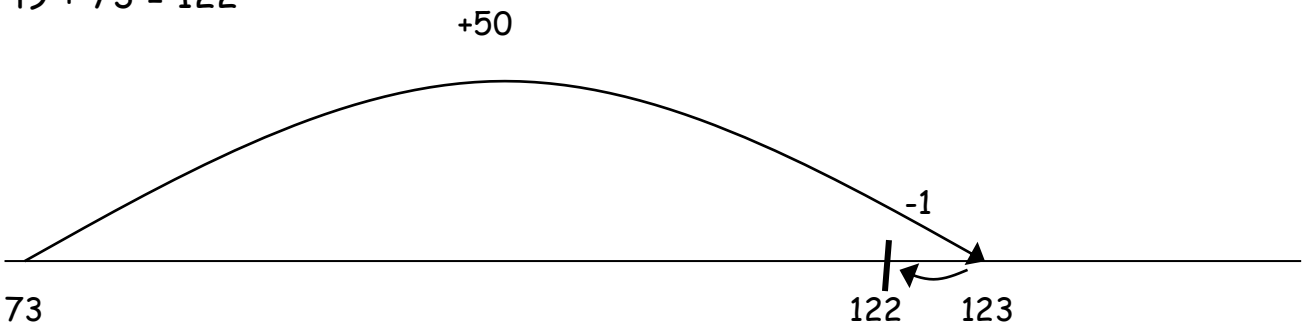
34 + 23 = 57



Bridging through ten can help children become more efficient.
37 + 15 = 52



Compensation (mental with jottings).
49 + 73 = 122



Children can also be shown that the number line can be used to add the units first, then the tens, but that this is not the most efficient way.

As well as number lines, children need to be developing a mental image of a 100 square to aid mental calculation skills:

Partitioning only the second number (mental with jottings)
34 + 27= ⟶ 34 + 20 + 7 = ⟶ 54 + 7= 61

Adjust/ use knowledge of partitioning
37 + 15 = 52 by partitioning as 35 + 2 + 15 or 35 + 10 + 5 + 2

They also need to use their knowledge of how numbers work:
Count on from the largest number irrespective of the order of the calculations, e.g.:
36 + 74 = ⟶ 74 + 30 + 6 = ⟶ 104 + 6 = 110

Use knowledge of inverse calculations, e.g.:
If 10 – 7 = 3 then 7 + 3 must be 10,
And applying this to calculating with larger numbers, e.g.:
If 70 – 7 = 63 then 63 + 7 must be 70.

WHEN READY:

Expanded method

Adding most significant digits first, and then moving to adding least significant digits.

```
  267
+  85
  200
   140  (60 + 80)
    12  ( 7 +  5)
  352
```

Moving to adding the least significant digits first in preparation for 'exchanging'.

```
   67
+  24
   11  ( 7 +  4)
   80  (60 + 20)
   91
```

```
  267
+  85
   12  ( 7 +  5)
  140  (60 + 80)
  200
  352
```

The use of diennes blocks and place value counters will help children to visualise the relationship between hundreds, tens and ones and understand why exchanging is necessary.

MOVING ONTO:

Columnar Addition

Children will begin to exchange, using the language of place value, e.g.:

```
   6¹25
+   48
   673
```

```
  ¹783
+  42
  825
```

```
  ¹3¹67
+  85
  452
```

Using similar methods, children will:

- add several numbers with different amounts of digits,
- begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds,
- know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p (this requires a big emphasis on place value),
- recognise and order negative numbers in horizontal and vertical number lines, including doing this in a real life context (e.g. temperature).

FINALLY

Children should extend the exchanging method to any size number, including decimals.

$$\begin{array}{r} {}^{1}5{}^{1}87 \\ +\ 4\,75 \\ \hline 10\,62 \end{array} \qquad \begin{array}{r} {}^{1}3{}^{1}5{}^{1}87 \\ +\ \ 6\,75 \\ \hline 4\,2\,6\,2 \end{array}$$

Using similar methods, children will:

- add two or more decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm (this requires a big emphasis on place value),
- more extended use of negative numbers

They should continue to use bar modelling to help them to solve word problems e.g.

4 children go to the cinema. They each pay £15. How much do they spend altogether?

?			
£15	£15	£15	£15

Or

During a race Chaniqwa has to swim 350m and run 1200m. What is the total distance she has to run and swim?

swim	run
350m	1200m

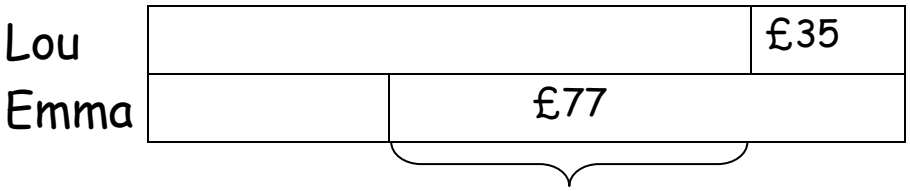
?

Bar modelling is particularly useful when solving multi-step word problems.

For example:

Lou and Emma have the same amount of money.
Lou spends £35 and Emma spends £77.
Lou now has 2½ times as much money left as Emma.
How much did they each start with?

Step 1 and 2

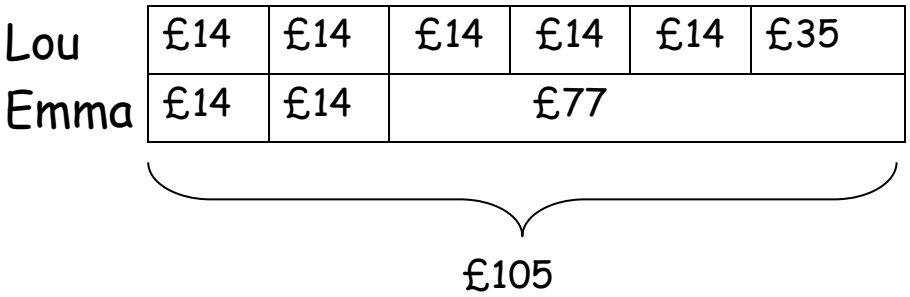


Calculations

£77 - £35 = £42

£42 ÷ 3 = £14

Step 3



£35 + £42 + £14 + £14

or £77 + £14 + £14=£105