

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

In Key Stage 1, the principal focus of mathematics teaching is to ensure that all children develop confidence and mental fluency with whole numbers, counting and place value. This will involve working with numerals, words and the four operations and will include the use of practical resources. By the end of year 2, children should know number bonds to 20 and use understanding of place value with numbers to at least 100.

In years 3 and 4, teaching will ensure that children become increasingly fluent with whole numbers, the four operations and place value. They will develop efficient written and mental methods and will work on calculations using increasingly large whole numbers.

The main focus for teaching in years 5 and 6 is to ensure that children extend their understanding of the number system and place value using large whole numbers (up to 10 000 000)

By the end of year 6, pupils should be fluent in written methods for all four operations and in working with fractions, decimals and percentages.

Children should not be made to go onto the next stage if:

- 1) They are not ready.
- 2) They are not confident.

Children should be encouraged to approximate their answers before calculating. They should also be encouraged to check their answers after calculation using an appropriate strategy. Children should be encouraged to consider if a mental calculation would be more accurate and efficient before using written methods. They are expected to use mathematical vocabulary and to read, spell and pronounce the language correctly.

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies that the children will use throughout their time in school. They are designed to be used alongside written methods.

Mental recall of subtraction facts

Number bonds for subtraction within 10 e.g.: $10 - 6 = 4$

Leading on to subtraction of multiples of 10: $4 - 3 = 1$; $40 - 30 = 10$

And: $20 - 17 = 3$ $17 - \square = 11$ $10 - \square = 2$

Subtracting by bridging through multiples of 10 using number bonds

$23 - 8 = 23 - 3 = 20$ (this can be called a 'safe number')

$20 - 5 = 15$ (number bonds to 8 used)

Find a small difference by counting up

$82 - 79 = 3$ (start at 79 and count up to 82 as it is the most efficient method)

Counting on or back in repeated steps of 1, 10, 100, 1000 and counting back in multiples of 10

$86 - 52 = 34$ (by counting back in tens/ multiples of 10 and then in ones)

$460 - 300 = 160$ (by counting back in hundreds)

Subtract 9/11 by subtracting 10 then +/- 1

$26 - 11 =$ $26 - 10 = 16 - 1 = 15$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$24 - 19 = 24 - 20 + 1 = 5$

$458 - 71 = 458 - 70 - 1 = 387$

Use the relationship between addition and subtraction

$5 + 4 = 9$

$4 + 5 = 9$

$9 - 4 = 5$

$9 - 5 = 4$

$36 + 19 = 55$

$19 + 36 = 55$

$55 - 19 = 36$

$55 - 36 = 19$

THE FOLLOWING ARE STRATEGIES THAT WE EXPECT THE MAJORITY OF CHILDREN TO USE AND UNDERSTAND.

INITIALLY:

In play and practical activities children should:

- Understand the meaning of 'more' and 'less',
- Know that a group of objects changes in quantity when something is added or taken away,
- Order a small set of numbers accurately
- Separate a group of 3 or 4 objects in different ways, beginning to recognize that the total is still the same
- Relate subtraction to taking away and counting how many more are left
- Take an object away and count how many are left.

THEN:

In practical activities (beginning to record when ready) children should:

- Use a range of vocabulary related to subtraction including knowing what 'subtraction' means,
- Sort/count objects,
- Use knowledge of number bonds to do calculations to 20,
- Begin to use - sign in practical games and activities,
- Compare two numbers and say which is more or less,
- Compare the amounts in two sets and say which is more or less,
- Say a number lying between two others, or before or after a given number,
- Find one more or less than a number from 1 to 20,
- Begin to understand doubling and halving,
- Remove a group of objects from a group and count how many are left,
- Takeaway a smaller number from a larger number and find out how many are left by counting back from the larger number.
- **Concrete modelling and handling with real objects**

This is the 'doing phase' where children handle real objects to solve problems.



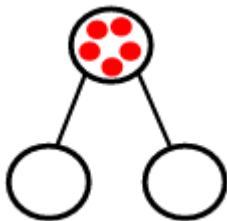
NEXT

Children use other objects to represent the real objects e.g. cubes instead of real apples.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.



Children can use the part-part-whole model:



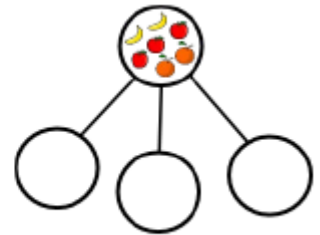
Here are seven pieces of fruit.



Put the fruit into a part whole model.
Complete the sentences.

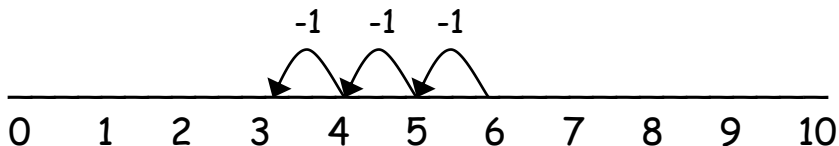
..... is the whole.

..... is a part, is a part and is a part.



They use number lines (vertical and horizontal) 100 squares, Numdrums and practical resources to support calculation. Teachers demonstrate the use of the numberline.

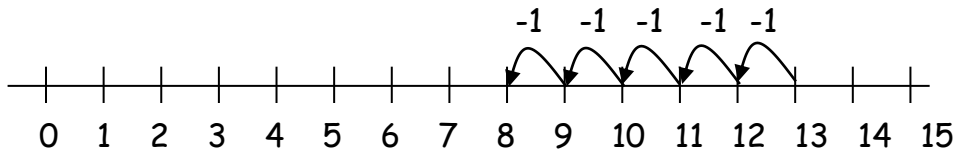
$$6 - 3 = 3$$



The number line should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.

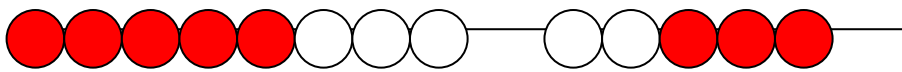
Children then begin to use numbered lines to support their own calculations, using it to count back in ones.

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



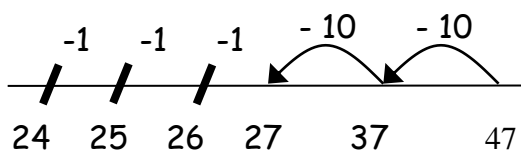
By this point children should be clear that subtraction cannot be done in any order.

AFTER THAT:

Children will begin to use empty number lines to support calculations by counting back.

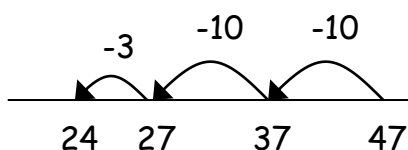
First counting back in ones, then counting back in tens and ones first by using numbers between 10 and 20 only, then by using numbers with multiples of 10

$$47 - 23 = 24$$



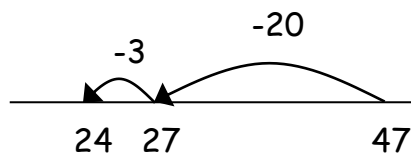
Children will then become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



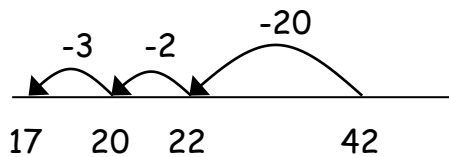
Followed by subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



They can also use bar modelling.

$17 - 13 = 4$ and $17 - 4 = 13$ and the inverse would be $13 + 4 = 17$.

17	
13	4

The bar model may be used slightly differently depending on the question.

I have 17 lego bricks. I used 13 bricks to build a wall. How many bricks do I have left?

17	
13	?

Or

I have 17 lego bricks. My friend has 4 bricks. How many more bricks than my friend do I have?

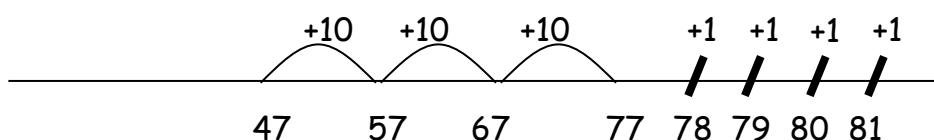
17	
?	4

When they are confident with this they will be taught to calculate by counting on, as this can be more efficient.

This strategy is especially effective if taught through the context of money.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

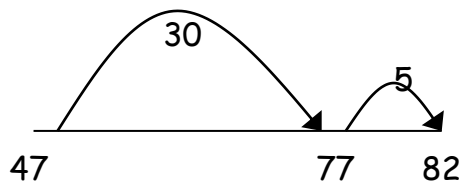
$$81 - 47 = 34$$



Children will become more efficient with counting on by:

- Subtracting the units in one jump,
- Subtracting the tens in one jump and the units in one jump,
- Bridging through ten.

$$82 - 47 = 35$$



Children also need to use their knowledge of how numbers work:

Use knowledge of inverse calculations, e.g.:

If $7 + 3 = 10$ then $10 - 7$ must be 3,

And applying this to calculating with larger numbers, e.g.:

If $63 + 7 = 70$ then $70 - 7$ must be 63.

WHEN READY:

Expanded partitioning and decomposition with no exchanging

This process could be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NB: When solving the calculation $89 - 57$, children should know that 57 DOES NOT EXIST AS AN AMOUNT it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

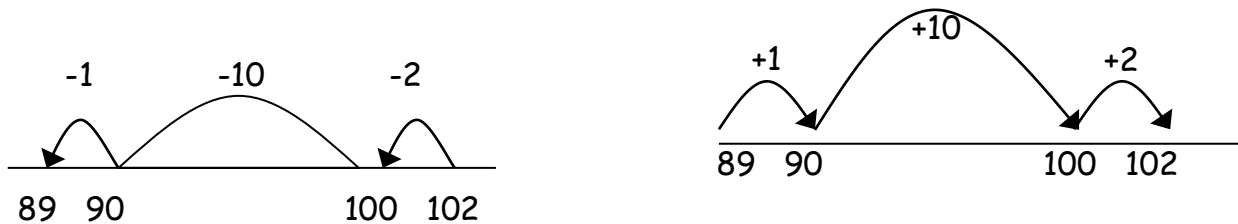
This is initially taught using examples that do not need to exchange.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 \\ 50 \\ \hline 30 \end{array} \begin{array}{r} 9 \\ 7 \\ \hline 2 \end{array} = 32$$

Children should know that units line up under units, tens under tens, and so on.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used as it is a more efficient way to calculate. This method, or the counting back method, may also be used for checking.

$$102 - 89 = 13$$



Children should also use rounding to check their answers to calculations.

MOVING ONTO:

Expanded partitioning and decomposition with exchanging

Single exchange T to U

$$\begin{array}{r} 384 \\ - 56 \\ \hline \end{array}$$

$$\begin{array}{r} 300 \quad 80 \quad 4 \\ - \quad \quad 50 \quad 6 \\ \hline \end{array} \longrightarrow \begin{array}{r} 300 \quad 70 \quad 14 \\ - \quad \quad 50 \quad 6 \\ \hline 300 \quad 20 \quad 8 = 328 \end{array} \quad (\text{adjust tens to units})$$

Recorded as:

$$\begin{array}{r} 300 + \overset{70}{\cancel{80}} + 14 \\ - \quad \quad 50 + 6 \\ \hline 300 + 20 + 8 = 328 \end{array}$$

The use of diennes blocks and place value counters will help children to visualise the relationship between hundreds, tens and ones and understand why exchanging is necessary.

Single exchange H to T

$$\begin{array}{r} 354 \\ - 182 \\ \hline \end{array}$$

$$300 \quad 50 \quad 4 \qquad 200 \quad 150 \quad 4 \quad (\text{adjust hundreds to tens})$$

$$\begin{array}{r}
 - \underline{100} \quad 80 \quad 2 \\
 \longrightarrow \\
 - \underline{100} \quad 80 \quad 2 \\
 \hline
 100 \quad 70 \quad 2 = 172
 \end{array}$$

Recorded as:

$$\begin{array}{r}
 \begin{array}{r}
 200 \\
 \cancel{300} \\
 - \underline{100} \\
 100
 \end{array}
 \quad
 \begin{array}{r}
 150 \\
 \cancel{50} \\
 - \underline{80} \\
 70
 \end{array}
 \quad
 \begin{array}{r}
 4 \\
 2 \\
 2
 \end{array}
 \\
 = 172
 \end{array}$$

Exchange H to T and T to U

$$\begin{array}{r}
 754 = \\
 - \underline{86}
 \end{array}$$

$$\begin{array}{r}
 \text{Step 1} \quad 700 \quad 50 \quad 4 \\
 - \quad \quad \underline{80 \quad 6}
 \end{array}$$

$$\begin{array}{r}
 \text{Step 2} \quad 700 \quad 40 \quad 14 \quad (\text{adjust from T to U}) \\
 - \quad \quad \underline{80 \quad 6}
 \end{array}$$

$$\begin{array}{r}
 \text{Step 3} \quad 600 \quad 140 \quad 14 \quad (\text{adjust from H to T}) \\
 - \quad \quad \underline{80 \quad 6} \\
 600 \quad 60 \quad 8 = 668
 \end{array}$$

This would be recorded by the children as:

$$\begin{array}{r}
 \begin{array}{r}
 600 \\
 \cancel{700} \\
 - \underline{80} \\
 600
 \end{array}
 \quad
 \begin{array}{r}
 140 \\
 \cancel{50} \\
 - \underline{80} \\
 60
 \end{array}
 \quad
 \begin{array}{r}
 14 \\
 6 \\
 8
 \end{array}
 \\
 = 668
 \end{array}$$

FINALLY:

As soon as children are secure in the expanded method they will move onto the contracted decomposition method:

$$\begin{array}{r}
 \begin{array}{r}
 6 \ 14 \ 1 \\
 \cancel{754} \\
 - \underline{86} \\
 668
 \end{array}
 \end{array}$$

Children should:

- be able to subtract numbers with different numbers of digits;
- using this method, children should also begin to find the difference between two three-digit sums of money, (could convert to whole numbers and back to £ and p after the calculation)
- know that decimal points should line up under each other (this requires a big emphasis on place value).

For example:

$$\begin{array}{r}
 \text{£}8.95 = \quad 8 \quad 0.9 \quad 0.05 \\
 \underline{-\text{£}4.38} \quad - \quad \underline{4 \quad 0.3 \quad 0.08} \\
 \\
 = \quad 8 \quad 0.8 \quad 0.15 \quad (\text{adjust from T to U}) \\
 \quad - \quad 4 \quad 0.3 \quad 0.08 \\
 \quad \quad 4 \quad 0.5 \quad 0.07 \\
 \\
 = \text{£}4.57
 \end{array}$$

leading to

$$\begin{array}{r}
 \quad \quad \quad 8 \quad 1 \\
 \quad \quad \quad \cancel{8} \quad 5 \\
 \quad \quad \quad - \quad 4.38 \\
 \hline
 \end{array}$$

Alternatively, children can set the amounts to whole numbers, i.e. 895 - 438 and convert to pounds after the calculation.

They should continue to use bar modelling to help them to solve word problems e.g.

253 children took part in an art competition. There are 134 girls. How many boys are there?

253 children	
134 girls	? boys

Calculation
253 - 134

Bar modelling is particularly useful when solving multi-step word problems.

For example:

A crate contains 3 identical metal balls.

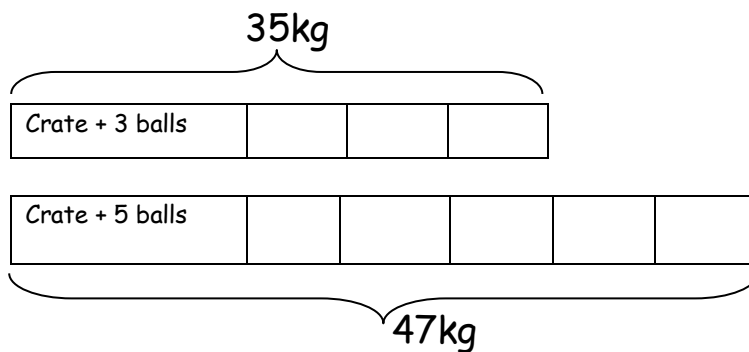
The mass of the crate and the balls is 35kg.

Two more identical balls are added to the crate.

The mass of the crate and balls is now 47kg.

What is the mass of the crate?

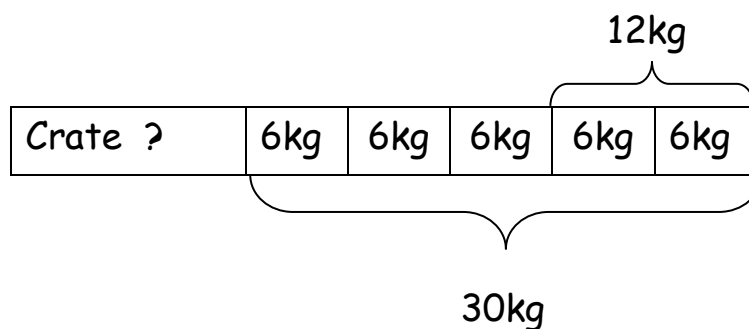
Step 1



Calculations

$$47\text{kg} - 35\text{kg} = 12\text{kg}$$

Steps 2,3,4



$$12\text{kg} \div 2 = 6\text{kg}$$

$$6\text{kg} \times 5 = 30\text{kg}$$

$$47\text{kg} - 30\text{kg} = 17\text{kg}$$

The mass of the crate is 17kg.