Year ⑥

Small Steps Guidance and Examples

Block 3 – Algebra

White Rose Maths

Year 6 – Yearly Overview

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number- Place Value		Number- Addition, Subtraction, Multiplication and Division				Fractions				Geometry- Position and Direction	Consolidation
Spring	Number- Decimals		Number- Percentages		Number- Algebra		Measurement Converting units	Measurement Perimeter, Area and Volume			Number- Ratio	Consolidation
Summer	Geometry- Properties of Shapes		Problem solving				Statistics		Investigations			Consolidation

Overview

Small Steps

Find a rule – one step

Find a rule – two step

Use an algebraic rule

Substitution

Formulae

Word Problems

Solve simple one step equations

Solve two step equations

Find pairs of values

Enumerate possibilities

NC Objectives

Use simple formulae.

Generate and describe linear number sequences.

Express missing number problems algebraically.

Find pairs of numbers that satisfy an equation with two unknowns.

Enumerate possibilities of combinations of two variables.

Find a Rule – One Step

Notes and Guidance

Children begin by exploring simple one step function machines. Explain that a one-step function is where they perform just one operation on a particular input value.

Children understand that for each number they put into a function there is an output.

Children should be able to write these one step functions as simple algebraic expressions. They should understand that we write simple functions such as $a \times 4$ as $4a$.

Mathematical Talk

What do you think one-step function means?

What examples of functions do you know? What do you think input and output means?

What is the output if ….?

What is the input if ….?

Work out the function if you know the following functions?

How many sets of input and output do you need to be able to work out the function? Explain your answers.
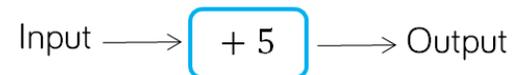
What is the algebraic rule for that function machine …?

Varied Fluency

1. Here is a function machine.

Input $\longrightarrow$ [× 4] $\longrightarrow$ Output

- What is the output if the input is 2?
- What is the output if he input is 7.2?
- What number went in if the output was 22?
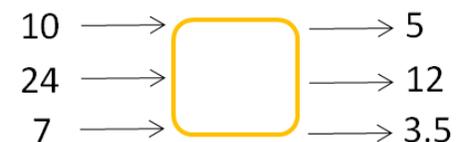- What is the output if the input is a? What about if you put x in?

2. Complete the table for the given function machine.

Input $\longrightarrow$ [+ 5] $\longrightarrow$ Output

Input	5	5.8	10	-3	-8				a	y
Output						9	169	0		

3. Write your function as an algebraic rule?
   Work out the functions

   $10 \longrightarrow$ [ ] $\longrightarrow 5$
   $24 \longrightarrow$ [ ] $\longrightarrow 12$
   $7 \longrightarrow$ [ ] $\longrightarrow 3.5$

Find a Rule – One Step

Reasoning and Problem Solving
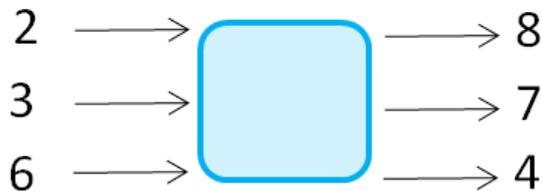
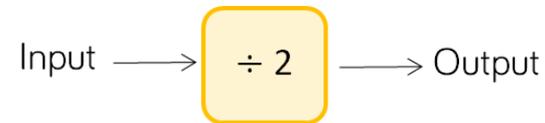|			
Meg has a one-step function machine. She puts in the number 6 and the number 18 comes out. 6 ⟶ ☐ ⟶ 18 What could the function be? How many different answers can you find?	The function could be +12, ×6, subtract from 24, divide by 1/3	Lucy is using the following function machine. Input ⟶ ÷ 2 ⟶ Output Lucy put a number into the machine. She puts the output back into the machine and gets out another number. The final answer is 2.5 What number did Lucy put in? Lucy has another function machine. • She puts a number 8 and gets an output. • She puts the output back into the machine. • The final output is -6 What could the function be?	10 Subtract 7 (- 7)
Giles puts in some numbers into a function machine. 2 ⟶ ☐ ⟶ 8 3 ⟶ ☐ ⟶ 7 6 ⟶ ☐ ⟶ 4 What is he output from the function when the input is 16?	The function is subtract from 10 so the output is -6		

Find a Rule – Two Step

Notes and Guidance

Children build on their knowledge of one-step functions to consider now two-step function machines. Discuss with children whether a function such as +5 and + 6 is a two-step function machine or whether it can be written as a one-step function. They look at strategies to find the functions, given a series of inputs and outputs. They do this by trial and error or by considering the pattern of differences.   Children record their input and output values in the form of table.

Mathematical Talk

How can you write + 5 followed by – 2 as a one-step function?
Do the functions have to be different?
If I switched around the functions, do you get the same answers?
What is the output if …? What is the input if …? How did you work it out?
What is the function machine if a is the input and $3a - 2$ comes out.
What method did you use to find a two-step function?

Varied Fluency

1 Here is a function machine.
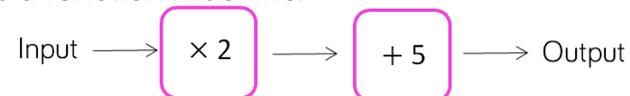
Input ⟶ ×2 ⟶ +5 ⟶ Output

- What is the output if the input is 5?
- What number went in if the output was 19
- What is the output if the input is a? What about if you put x in?

2 Complete the table for the given function machine.

Input ⟶ ×3 ⟶ − 4 ⟶ Output

Input	1	2	3	4	5
Output					

- What patterns do you notice in the outputs?
- What is the input if 20 comes out? How did you work it out?
- What is the algebraic function for this function?

3 What does the function machine look like for each of these algebraic expressions?

$$a \times 3 + 2 \qquad 5a - 4 \qquad (a + 3) \times 2$$

Find a Rule – Two Step

Reasoning and Problem Solving

David has a two function machines.

Input $\longrightarrow$ $+5$ $\longrightarrow$ $\times 2$ $\longrightarrow$ Output

Input $\longrightarrow$ $\times 2$ $\longrightarrow$ $+5$ $\longrightarrow$ Output
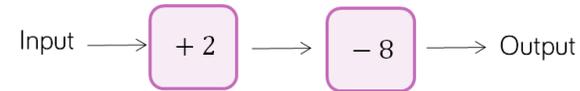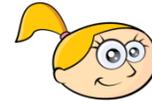
He says,

> The function machines will give the same answer.

Is David correct?
Is there an input that will give the same output for both machines?

No they do not give the same answer. Children can just put any number in to show this.

Lola has the following function machine.

Input $\longrightarrow$ $+2$ $\longrightarrow$ -8 $\longrightarrow$ Output

Explain how this can be written as a single function machine.

Write is as -6

Use an Algebraic Rule

Notes and Guidance

Children have now met one-step and two-step function machines and have formed expressions from these machines. Children are now presented with abstract algebraic expressions and have to work out the one-step or two-step rules.

They again work out input and output values given the rule, not realising at this stage that they are doing formal substitution and solving equations. Children need to be able to recognise standard simplified versions of the rules.

Mathematical Talk

What expressions can be formed from this function machine?

What would the function machine look like for this rule/expression?

How can you write $a \times 3 + 5$ differently?

Are $2a + 6$ and $6 + 2a$ the same? Explain

Varied Fluency

1. What algebraic rules come from the following function machines?

$Input \longrightarrow \boxed{\times 5} \longrightarrow Output$

$Input \longrightarrow \boxed{\times 2} \longrightarrow \boxed{-5} \longrightarrow Output$

What is the output when 7 is input into each function machine?

2. What does the function machine look like for each of these algebraic expressions?

$a \times 4$	$a + 10$	$a - 10$
$a \div 2$	$3a$	$a \div 5$
$a \times 5 + 3$	$3a - 1$	$(a + 3) \times 2$

What is the output when 7 is substituted into each of these functions?
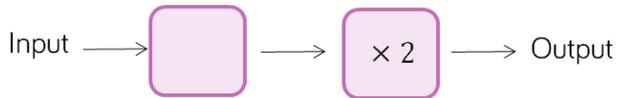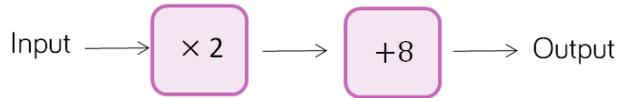
3. Here is an expression that has come from a function machine: Output = $4a + 3$
   - What is the output if the input, a, is 7?
   - What is the output if he input is 2.5?
   - What is the input if the output is 63?

Use an Algebraic Rule

Reasoning and Problem Solving

These two function machines give the same answer.

Input $\longrightarrow$ ×2 $\longrightarrow$ +8 $\longrightarrow$ Output

Input $\longrightarrow$ [ ] $\longrightarrow$ ×2 $\longrightarrow$ Output

What is missing part of the function?
Can you explain why using base 10?
Using algebraic expressions?

This function machine gives the same output for every input.
For example if the input is 5 then the output is 5 and so on.

Input $\longrightarrow$ ×2 $\longrightarrow$ [ ] $\longrightarrow$ Output

What is the missing part of the function?
What other pairs of functions can you think that will do the same?

+ 4
Because a x 2 + 8
is the same as
(a + 4) x 2

÷2
Functions that are the inverse of each other.

Complete the table of values for the following function.

$$5 + 2a$$

Input	5	7		
Output			75	30

15, 19, 35, 12.5

Substitution

Notes and Guidance

Children substitute into simple expressions and equations to find a particular value.

They have already experienced substitution in a less formal way and allowing children to see the link between this and formal substitution will help it feel less abstract.

Mathematical Talk

Which letter represents the star? Which letter represents the heart?
Would it still be correct if it changed to $a + b + c$?

What do you notice about your final answer in question 2 and your first answer in question 3?

What does it mean when a number is next to a letter?

Varied Fluency

1. If ⭐ $= 7$ ❤ $= 5$ what is the value of:

   ⭐ $+$ ❤ $+$ ❤

   What is the same and what is different about this question?
   If $a = 7$ and $b = 5$ what is the value of:
   $$a + b + b$$

2. Substitute into the following expressions when,

   $$w = 3 \quad x = 5 \quad y = 2.5$$

   - $w + 10$
   - $w + x$
   - $y - w$
   - $w + x + y$
   - $w - x - y$
   - $y + y + y$

3. Substitute into the following expressions when,

   $$w = 10 \quad x = \frac{1}{4} \quad y = 2.5$$

   - $3y$
   - wx
   - $12 + 8.8w$
   - $x \times (w + 2y)$

Substitution

Reasoning and Problem Solving

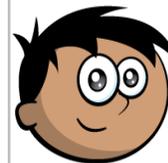Here are two equations.

$$p = 2a + 5$$

$$c = 10 - p$$

Find the value of c when $a = 10$

$c = -15$

$$x = 2c + 6$$

Joe says,

$x = 12$ because c must be equal to 3 because it's the 3rd letter in the alphabet

Is Joe correct?

Derek says,

When $c = 5$ the answer is 31

Is Derek correct?

No Joe is incorrect. c could have any value.

No Derek is incorrect, he has just put the 2 and 5 together to make 25 instead of multiplying them.

Formulae

Notes and Guidance

Children substitute into familiar formulae such as the formula for area and volume.

They also use simple formulae to work out values of everyday activities such as the cost of a taxi or the amount of medicine to take given a person's age.

Mathematical Talk

What tells you something is a formula?

In the formula $C = £1.50 + 0.3m$ what do you think the 'C' stands for?
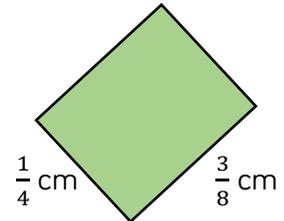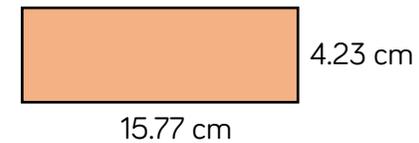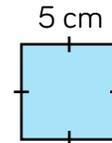What do you think the m stands for?

Varied Fluency

1 Tick the formula.

$$P = 2(l + w) \qquad 3d + 5 \qquad 20 = 3x - 2$$

Explain why the other two are not formulae.

2 Substitute into $P = 2(l + w)$ to find the perimeter of the following rectangles and squares.

5 cm

4.23 cm

15.77 cm

$\frac{1}{4}$ cm $\frac{3}{8}$ cm

Use the formula for area of a rectangle to also find the area.

3 This is the formula to work out the cost of a taxi.

$$C = 1.50 + 0.3m$$

$m = $ number of miles travelled.
Work out the cost of the taxi when it travels 12 miles.

Formulae

Reasoning and Problem Solving

Joe and Nadia are using the following formula to work out what they should charge for four hours' cleaning.

Cost in pounds = 20 + 10 × number of hours

Joe writes down £60
Nadia writes down £120

Who do you agree with?
Why?

Joe is correct as multiplication should be performed first.

Nadia has not used the order of operations.

The rule for making scones is use 4 times as much flour (f) as butter (b).

Which is the correct formula to represent this?

A
$$f = \dfrac{b}{4}$$

B
$$f = 4b$$

C
$$f = b + 4$$

D
$$4f = b$$

Explain why the others are incorrect.

B is correct.

Word Problems

Notes and Guidance

Children begin to start thinking about solving equations through worded problems.

This helps children see a reason for solving an equation and gives them something to relate more abstract equations to.

Ensure to use concrete materials when first introducing this concept.

Mathematical Talk

What does the cube represent?
What do the counters represent?

Can you think of your own 'think of a number' problems?

Why are the questions in Q3 more difficult to represent using concrete materials?

Varied Fluency

① Here is a word problem represented with concrete resources and algebra.

Words	Concrete	Algebra
I think of a number	🟦	x
Add 3	🟦🔴🔴🔴	$x + 3$
My answer is 5	🟦🔴🔴🔴 = 🔴🔴🔴🔴🔴	$x + 3 = 5$

Can you complete this table?

Words	Concrete	Algebra
I think of a number		
Add 1		
My answer is 8		

② Use concrete materials to represent these equations.

$$w + 4 = 7 \qquad 10 = 2 + t \qquad 3 + x = 9$$

③ Write the algebra to match the sentences.
- I think of a number, subtract 17, my answer is 20
- I think of a number, multiply it by 5, my answer is 45

Word Problems

Reasoning and Problem Solving

Jane thinks of a number, she adds 7 and divides her answer by 2

Mike thinks of a number, he multiples by 3 and subtracts 4

Jane and Mike think of the same number.
Jane's answer is 9
What is Mike's answer?

Jane and Mike think of the same number again and they both get the same answer.

Use trial and error to find the number they were thinking of

They both think of 11, therefore Mike's answer is 29

They think of 3 and the answer they both get is 5

Kira spends 92p on yoyos and sweets

She buys y yoyos costing 11p and s sweets costing 4p.

Can you write an equation to represent what Kira has bought?

How many yoyos and sweets could Kira have bought?

Can you write a word problem to describe this equation?

$$74 = 15t + 2m$$

$92 = 11y + 4s$

She could have bought 1 sweet and 8 yoyos or 4 yoyos and 12 sweets

One Step Equations

Notes and Guidance

Children solve simple one step equations involving the four operations.

Children should explore and build on the use of concrete materials such as cubes, counters and cups.

It is recommended that children learn to solve equations using a balancing method and the use of inverse operations.

Mathematical Talk

Can you make some of your own equations using cups and counters for a friend to solve?

Why do you think the equation is set up on a balance? What does the balance represent? How does this help you solve the equation?

What is the same and what is different about each bar model?

Varied Fluency

1 What's in the cup?
Write down and solve the equation represented by the diagram.
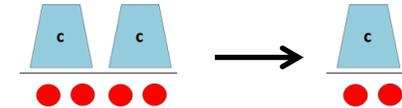
2 Solve the equation represented on the scales.
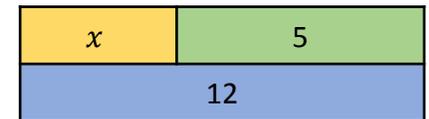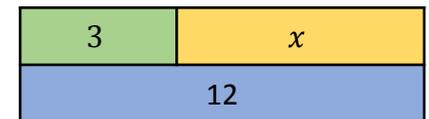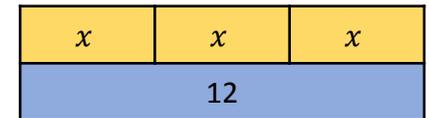Can you draw a diagram to go with the next step?
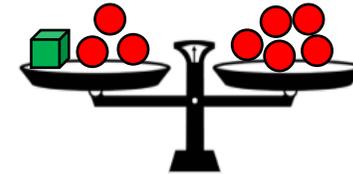
3 Match each equation to the correct bar model then solve.

$x + 5 = 12$

$3x = 12$

$12 = 3 + x$

x	x	x
12		

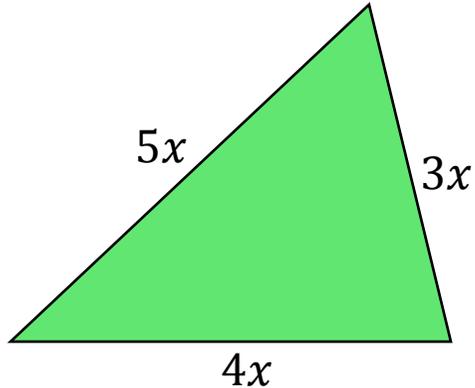| 3	x
12	

x	5
12	

One Step Equations

Reasoning and Problem Solving

The perimeter of the triangle is 216 cm.

Work out each length of the triangle by forming and solving an equation then substituting your answer.

$12x = 216$
$x = 18$

$5 \times 18 = 90$
$3 \times 18 = 54$
$4 \times 18 = 72$

- Hannah is 8 years old
- Jack is 13 years old
- Grandma is $x + 12$ years old.
- The sum of their ages is 100

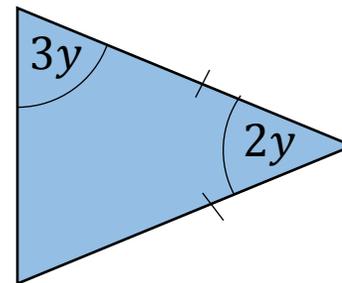Form and solve an equation to work out how old Grandma is.

$8 + 13 + x + 12 = 100$

$33 + x = 100$

$x = 77$

Grandma is 77 years old.

What is the size of the smallest angle in this isosceles triangle?

How can you check your answer?

$8y = 180$
$y = 22.5$

Smallest angle = 45°

Check by working them all out and see if they add to 180°

Two Step Equations

Notes and Guidance

Children progress from solving equations that require one step to equations that require two steps.

Children should think of each equation as balance and solve it through doing the same thing to each side of the equation.
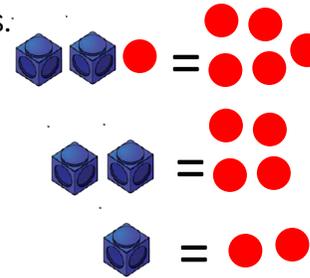
Mathematical Talk

Why do you have to do the same to each side of the equation?

Why subtract 1? What does this do to the left hand side of the equation?

Does the order the equation is written in matter?

Varied Fluency

1. Here is each step of an equation represented with concrete resources.

$$2x + 1 = 5$$
$$-1 \qquad\qquad -1$$
$$2x = 4$$
$$\div 2 \qquad\qquad \div 2$$
$$x = 2$$

Use this method to solve:

$$4y + 2 = 6 \qquad 9 = 2x + 5 \qquad 1 + 5a = 16$$

2. Solve the following equations.

$$3y + 5 = 26 \qquad\qquad 10 = 17 + 2x$$

$$0.5w - 1 = 0 \qquad\qquad 2q - \frac{1}{6} = 6 - \frac{1}{6}$$

$$114 = \frac{y}{5} + 99 \qquad\qquad 10 - 2x = 4$$

Two Step Equations

Reasoning and Problem Solving

The length of a rectangle is $2x + 3$
The width of the same rectangle is $x - 2$
The perimeter is 17 cm

Find the area of the rectangle.

$6x + 2 = 17$
$6x = 15$
$x = 2.5$
Length = 8 cm
Width = 0.5 cm
Area = 4 cm²

Katy has some algebra expression cards.

$y + 4$ $2y$ $3y - 1$

The mean of the cards is 19
Work out the value of each card.

$6y + 3 = 57$
$6y = 54$
$y = 9$

Card values:
13
18
26

The diagram shows a quadrilateral ABCD.

The perimeter of the quadrilateral is 80 cm.

B
21 cm
$4y + 1$
C
A
$5y + 1$
D
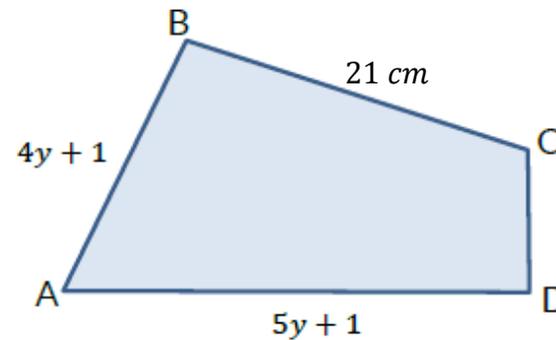
AB is the same length as BC.

Find the length of CD.

$4y + 1 = 21$
$4y = 20$
$y = 5$
AB = 21 cm
BC = 21 cm
AD = 26 cm
CD = 12 cm

Find Pairs of Values

Notes and Guidance

Children will use their understanding of how to solve an equation and apply this knowledge to finding the possible values of a pair.

Mathematical Talk

What is the question asking you to do?

How many possible answers are there? Convince me you have them all.

What do you notice about the values of a and b?

Varied Fluency

1. a and b are variables:

$$a + b = 6$$

Find 5 different possible values for a and b.

a	b

2. X and Y are whole numbers.
   - X is a one digit odd number.
   - Y is a two digit even number.
   - $X + Y = 25$

   Find all the possible pairs of numbers that satisfy the equation.

3.
$$a \times b = 48$$

What is the value of a and b?
How many different ways can you find?

Find Pairs of Values

Reasoning and Problem Solving

a, b and c are integers between -5 and 5

$$a - b = -3$$
$$b + c = 3$$

Find the values of a, b and c
How many different possibilities can you find?

Use the possible values to complete the equation:

$$a + c = \boxed{}$$

Possible answers:

a = -5   b = -2
c = 5
a = -4   b = -1
c = 4
a = -3   b = 0
c = 3

a + c = 0

---

x and y are both positive whole numbers.

$$\frac{x}{y} = 4$$

Jade says,

> x will always be a multiple of 4

Simon says,

> y will always be a factor of 4

Who is correct?
Prove it!

Possible answer:
Jade is correct as x will always have to divide into 4 equal parts. E.g, 32 ÷ 8 = 4, 16 ÷ 4 = 4
Simon is incorrect.
40 ÷ 10 = 4 and 10 is not a factor of 4

Enumerate Possibilities

Notes and Guidance

Children see they can enumerate possibilities (list of possibilities)

They need to use number properties efficiently to satisfy a specific criteria that is set.

Mathematical Talk

What does $2a$ mean? (2 multiplied by an unknown number)
What is the greatest/smallest number 'a' can be?

What strategy did you use to find the value of 'b'?

Can you draw a bar model to represent the following equations:
$3f + g = 20$
$7a + 3b = 40$
What could the letters represent?

Varied Fluency

1 In this equation, a and b are both whole numbers which are less than 12.

$$2a = b$$

Write the calculations that would show all the possible values for a and b.

2 Use the equation to fill in the missing values in the table below.

$$7x + 4 = y$$

Value of x	Value of y

3 $2g + w = 15$
Write down all the possible values for g and w, show each of them in a bar model.

15		
g	g	w

Enumerate Possibilities

Reasoning and Problem Solving

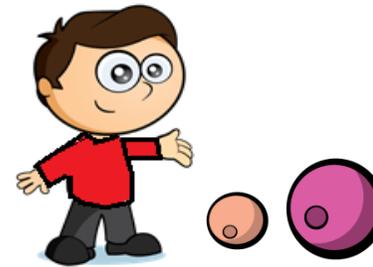$ab + b = 18$

Deanna says,

> *a* and *b* must both be odd numbers

Is Deanna correct?
Prove it.

Possible answer:

Deanna is incorrect. Children may give examples to prove Deanna is correct. For example, 5 × 3 + 3 = 18.
But there are also examples to prove she is incorrect. E.g. 2 × 6 + 6 = 18 where a and b are both even.

Large beads cost 5p and small beads cost 4p

Mr Smith has 79p to spend on beads.

How many different combinations of small and large beads can Mr Smith buy?

Can you write expressions that show all the solutions?

Possible answers:

$11l + 6s$
$7l + 11s$
$15l + s$
$3l + 16s$